AN INVENTORY MODEL FOR DECAYING ITEMS WITH RAMP TYPE DEMAND UNDER LEARNING EFFECT

Jayshree* and S.R. Singh**

*Department of Mathematics, C.C.S. University Meerut, jayashri000@gmail.com
**Department of Mathematics, C. C. S. University Meerut, shivrajpundir@gmail.com

ABSTRACT: This paper develops a generalized inventory model for decaying items with ramp type demand rate under learning effect. Keeping in view the concern about that the performance of a system engaged in a repetitive process improves with time, this phenomenon is referred to as the 'learning effect'. It would be unethical if the effect of learning is not considered while determining the optimal inventory policy. Moreover, deterioration of items is a general phenomenon; due to this phenomenon the item may not serve the purpose after a period of time and will have to be discarded as it cannot be used to satisfy the future demand of customers. Thus for managing inventory in a realistic scenario, the effect of deterioration cannot be disregarded. In this paper we used ramp type function of time for demand. By analyzing the model, an efficient solution procedure is proposed to determine the optimal costs for two different cases. Finally, numerical examples are provided to illustrate the theoretical results and a sensitivity analysis of the major parameters with respect to the stability of optimal solution is also carried out.

KEYWORDS: ramp type demand, learning, deterioration, unit production cost, no shortage.

INTRODUCTION

In the existing literature many classical inventory models assume that demand is constant. In present marketing environment, few items follow constant demand. But researches commonly use a time varying demand pattern which is used to keep the track of sales of items in different time phase of the replenishment cycle. Donaldson [1977] extended the constant demand to linear time dependent demand model analytically with finite time horizon. Thereafter numerous research works have been equipped with time-varying demand patterns into inventory models such as Hariga [1996]. In the literature mainly two types of time-varying demand rate have been used (1) linearly time dependent (2) exponentially time dependent But in practice, it is not possible i.e. demand increase continuously or decrease continuously. The demands of fashionable goods increase up to a certain level and after that become steady. Such type of demand functions firstly proposed by Hill [1995] who modeled, demand function increases linearly at the beginning and then after maturation it becomes constant, a stable stage at the end of the inventory cycle and termed it as "ramp-type" time-depended demand pattern.

The inventory models with ramp-type demand rate also established by Wu et al. [2008], Cheng & Wang [2009], Singh et al [2007]. In above mentioned paper, to obtain optimality needs to find the time point, when the inventory level reaches zero. Resultantly, the following two cases should be examined: (I) when the demand rate is stabilized, the time point occurs before that point (2) when the demand is stabilized after that the time point occurs. Deng et al. [2007] developed the inventory model of Wu & Ouyang [2000] by exploring these two cases. Skouri et al [2011] extended the work of Deng et al. [2007]. Recently Manna et al. [2016] studies an EOQ model with ramp type demand rate, constant deterioration rate and unit production cost.

Another phenomenon that affects the inventory systems is deterioration. We assumed the goods in inventory to have an infinite lifespan or presumed to be perfect throughout the business cycle. But in reality, there are some items which subjected to several risks such as pilferage, breakage, evaporation, spoilage and vaporization. Deterioration means worsening of products. It is a natural phenomenon in our daily life. Food items, pharmaceuticals, chemicals, volatile liquid, blood etc that deteriorates during their normal stage period. So, researchers have been progressively modifying the existing models for deteriorating items so as to make them more practical and realistic. Ghare and Schrader [1963]

developed in EOQ model for deteriorating items by assumed exponential decay. Covert and Philip [1973] devised the EOQ model with the assumption of weibull distribution deterioration. Goyal & Giri [2001] gave a comprehensive review of research on deteriorating items. Further, research in this field is summarized by different survey papers Goyal et al. [2013], Manna and Chaudhary [2006].

It is a human phenomenon that the performance of a person, group of persons, or an organization, engaged in a repetitive task; they learn how to perform efficiently and quickly i.e. they improve with time. Hence a reduction in the cost or the required for producing each unit is known as "learning phenomenon". In practice the effective use of machines and tools are usually increased by repetition and Wright suggested the learning curve is defined by a power function. The strategic importance of learning effects can be found in the review articles by Yadav et al. (2011), Kumar et al. (2013).

Manna et al. [2016] extended the work of Manna and Chaudhary [2006]. This paper contained ramp type demand rate, constant deterioration and unit production cost. Incorporating all the above facts, this study develops an inventory model under learning phenomenon where (a) the demand is stabilized after and before the production stopping time consequently (b) Deterioration considers as a function of time.

Further this study is organized as follows section 2 defines the assumptions and notations. Section 3 provides mathematical formulation of the model. Where optimal cost are determined for two different cases. At last illustrates the proposed inventory model with numerical examples along with sensitivity. Finally, observations and conclusions are made.

ASSUMPTIONS AND NOTATIONS

The following notations and assumptions are considered to develop the inventory model

Notations

c_1 – Holding cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $c_{01}+\frac{c'_1}{n^{\alpha 2}}$, $\alpha_2 > 0$

c_3 – Deterioration cost per order is partly constant and partly decreases in each cycle due to learning effect and defined as $c_{03}+\frac{c'_3}{n^{\alpha 2}}$, $\alpha_2 > 0$

C – Total average cost for a production cycle

Assumptions

(1) Demand rate in ramp type function of time, i.e. demand rate D= f (t) is assumed to be a ramp type function of time f (t)= D_0[t-(t-μ) H(t-μ)] , $D_0 > 0$ and H(t) is a Heaviside's function:

$$H(t-\mu) = \begin{cases} 1 & if\ t \geq \mu \\ 0 & if\ t < \mu \end{cases}$$

(2) Deterioration varies unit time and it is function of the time, i.e. θt , where t denote time of deterioration .

(3) Lead time is zero.

(4) Costs are considered under learning phenomenon.

(5) K=β f (t) is the production rate where β (> 1) is a constant.

The unit production cost $v = \alpha_1 R^{-\gamma}$ where $\alpha_1 > 0, \gamma > 0$ and $\gamma \neq 2$.

α_1 is obviously positive since v and R are both non-negative. Also higher demands result in lower unit cost of production. This implies that v and R are inversely related and hence, must be non-negative i.e. positive.

Now,

$$\frac{dV}{dR} = -\alpha_1 \gamma R^{-(\gamma+1)} < 0.$$

$$\frac{d^2V}{dR^2} = \alpha_1 \gamma(\gamma + 1) R^{-(\gamma+2)} > 0.$$

Thus, marginal unit cost of production is an increasing function of R. These results imply that, as the demand rate increases, the unit cost of production decreases at an increasing rate. Due to this reason, the manufacture is encouraged to produce more as the demand for the item increases. The necessity of restriction $\gamma \neq 2$ asises from the nature of the solution of the problems.

MATHEMATICAL FORMULATION OF THE MODEL

Case 1 ($\mu \leq t_1 \leq t_2$)

The stock level initially is zero. Production starts just after t=0. When the stock attains a level s at time t= t_1, then the production stops at that time. The time point μ occurs before the point t=t_1, where demand is stabilized after that the inventory level diminishes due to both demand and deterioration ultimately falls to zero at time t = t_2. Then, the cycle repeats.

Let I (t) be the inventory level of the system at any time t ($0 \le t \le t_2$). The differential equations governing the system in the interval [0, t_2] are given by

$$\frac{dI(t)}{dt} + \theta t I(t) = K - f(t), \qquad 0 \le t \le \mu \tag{1}$$

With the condition I(0)=0;

$$\frac{dI(t)}{dt} + \theta t I(t) = K - f(t), \qquad \mu \le t \le t_1 \tag{2}$$

With the condition $I(t_1) = s$;

$$\frac{dI(t)}{dt} + \theta t I(t) = -f(t), \qquad t_1 \le t \le t_2 \tag{3}$$

With the condition $I(t_1) = s$, $I(t_2) = 0$;

Using ramp type function f (t) equation (1),(2),(3) respectively

$$\frac{dI(t)}{dt} + \theta t I(t) = (\beta - 1)D_0 t, \qquad 0 \le t \le \mu \tag{4}$$

With the condition I(0)=0;

$$\frac{dI(t)}{dt} + \theta t I(t) = (\beta - 1)D_0\, \mu, \qquad \mu \le t \le t_1 \tag{5}$$

With the condition $I(t_1) = s$;

$$\frac{dI(t)}{dt} + \theta t I(t) = -D_0 \mu, \qquad t_1 \le t \le t_2 \tag{6}$$

With the conditions of $I(t_1) = s, I(t_2) = 0$;

The Solution of equation (4) is given by

$$I(t)\, e^{\frac{\theta t^2}{2}} = (\beta - 1)D_0 \left(\frac{t^2}{2} + \frac{\theta t^4}{8} + \frac{\theta^2 t^6}{24}\right) + c \tag{7}$$

using initial condition $I(0) = 0$, in equation (7), we get

$$c = 0 \tag{8}$$

$$I(t) = (\beta - 1)D_0 \left(\frac{t^2}{2} + \frac{\theta t^4}{8} + \frac{\theta^2 t^6}{24}\right) e^{\frac{\theta t^2}{2}} \tag{9}$$

$$\frac{dI(t)}{dt} + \theta t I(t) = (\beta - 1)D_0 \mu$$

$$\int_\mu^t d[I(t)e^{\frac{\theta t^2}{2}}] = (\beta - 1)D_0 \mu \int_\mu^t e^{\frac{\theta t^2}{2}} dt$$

$$I(t)e^{\frac{\theta t^2}{2}} = (\beta - 1)D_0 \mu \left[(t - \mu) + \frac{\mu}{2} + \frac{\theta \mu^3}{8} + \frac{\theta}{6}(t^3 - \mu^3) + \frac{\theta^2 \mu^5}{24}\right]$$

$$I(t) = (\beta - 1)D_0 \mu e^{-\frac{\theta t^2}{2}} \left[t - \frac{\mu}{2} + \frac{\theta t^3}{6} - \frac{\theta \mu^3}{24} + \frac{\theta^2 t^5}{20} - \frac{\theta^2 \mu^5}{120}\right], \mu \le t \le t_1 \tag{10}$$

The solution of equation (6) is given by

$$I(t) = -D_0 \mu (t + \frac{\theta t^3}{6} + \frac{\theta^2 t^5}{20}) e^{-\frac{\theta t^2}{2}} + D_0 \mu \left(t_1 + \frac{\theta t_1^3}{6} + \frac{\theta^2 t_1^5}{20}\right) e^{-\frac{\theta t^2}{2}} + s^{\theta \frac{(t_1^2 - t^2)}{2}} \tag{11}$$

using initial condition $I(t_2) = 0$ in equation (11), we get

$$s = D_0 \mu\, e^{-\frac{\theta t_1^2}{2}} \left[(t_2 - t_1) + \frac{\theta}{6}(t_2^3 - t_1^3) + \frac{\theta^2}{20}(t_2^5 - t_1^5)\right] \tag{12}$$

Substituting s in equation (11), we get

$$I(t) = D_0 \mu \left[(t_1 - t) + \frac{\theta}{6}(t_1^3 - t^3) + \frac{\theta^2}{20}(t_1^5 - t^5)\right] e^{-\frac{\theta t^2}{2}} + D_0 \mu\, e^{-\frac{\theta t_1^2}{2}} \left[(t_2 - t_1) + \frac{\theta}{6}(t_2^3 - t_1^3) + \frac{\theta^2}{20}(t_2^5 - t_1^5)\right] e^{\frac{\theta(t_1^2 - t^2)}{2}}$$

$$I(t) = D_0 \mu\, e^{-\frac{\theta t^2}{2}} \left[(t_2 - t) + \frac{\theta}{6}(t_2^3 - t^3) + \frac{\theta^2}{20}(t_2^5 - t^5)\right], \; t_1 \le t \le t_2 \tag{13}$$

The total inventory in $[0, t_2]$is

$$\int_0^{t_2} I(t)dt = \int_0^{\mu} I(t)dt + \int_\mu^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt$$

$$\int_0^{\mu} I(t)dt = (\beta - 1)D_0 \left[\frac{\mu^3}{6} - \frac{\theta \mu^5}{40} - \frac{\theta^2 \mu^7}{48 \times 7}\right]$$

$$\int_\mu^{t_1} I(t)dt = (\beta - 1)D_0 \mu [\frac{t_1^2}{2} - \frac{\mu t_1}{2} - \frac{\theta t_1^4}{12} + \frac{\theta \mu^4}{24} - \frac{\theta \mu^3 t_1}{24} - \frac{\theta^2 t_1^6}{180} + \frac{5\theta^2 \mu^6}{720} + \frac{\theta \mu {t_1}^3}{12} + \frac{\theta^2 t_1^3 \mu^3}{144} - \frac{\theta^2 \mu^5 t_1}{120}]$$

$$\int_{t_1}^{t_2} I(t)dt = D_0\mu\left[\frac{t_2^2}{2} + \frac{t_1^2}{2} - t_2t_1 + \frac{\theta t_2^4}{12} - \frac{\theta t_2^3 t_1}{6} - \frac{\theta t_1^4}{12} + \frac{\theta^2 t_2^6}{36} - \frac{\theta^2 t_1^6}{180} - \frac{\theta^2 t_2^5 t_1}{20} + \frac{\theta t_2 t_1^3}{6} + \frac{\theta^2 t_2^3 t_1^3}{36}\right]$$

$$\int_0^{t_2} I(t)dt = (\beta-1)D_0\left[\frac{\mu^3}{6} - \frac{\theta\mu^5}{40} - \frac{\theta^2\mu^7}{48\times 7}\right] + (\beta-1)D_0\mu\left[\frac{{t_1}^2}{2} - \frac{\mu t_1}{2} - \frac{\theta {t_1}^4}{12} + \frac{\theta\mu^4}{24} - \frac{\theta\mu^3 t_1}{24} - \frac{\theta^2 {t_1}^6}{180} + \frac{5\theta^2\mu^6}{720} + \frac{\theta\mu {t_1}^3}{12} + \frac{\theta^2 {t_1}^3\mu^3}{144} - \frac{\theta^2\mu {t_1}^5}{40}\right] + D_0\mu\left[\frac{{t_2}^2}{2} + \frac{{t_1}^2}{2} - t_2t_1 + \frac{\theta {t_2}^4}{12} - \frac{\theta {t_2}^3 t_1}{6} - \frac{\theta {t_1}^4}{12} + \frac{\theta^2 {t_2}^6}{36} - \frac{\theta^2 {t_1}^6}{180} - \frac{\theta^2 {t_2}^5 t_1}{20} + \frac{\theta t_2 {t_1}^3}{6} + \frac{\theta^2 {t_2}^3 {t_1}^3}{36}\right] \quad (14)$$

Total no of deteriorated items in $[0, t_2]$ is given by

Production in $[0, \mu]$ + Production in $[\mu, t_1]$ − Demand in $[0, \mu]$ − Demand in $[\mu, t_1]$

$$= \beta\int_0^{\mu} D_0 t\, dt + \beta\int_{\mu}^{t_1} D_0\mu\, dt - \int_0^{\mu} D_0\, t\, dt - \int_{\mu}^{t_2} D_0\, \mu\, dt$$

$$= \frac{1}{2}D_0\beta\mu\,(2t_1 - \mu) - \frac{1}{2}D_0\mu(2t_2 - \mu) \quad (15)$$

The cost of Production in $[u, u + du]$is

$$Kv\, du = \beta f(t)\alpha_1 R^{-\gamma} du$$

$$= \beta R\alpha_1 R^{-\gamma}\, du$$

$$= \alpha_1\beta R^{1-\gamma} du = \frac{\alpha_1\beta}{R^{\gamma-1}} du \quad (16)$$

Hence the Production cost in $[0, t_1]$is

$$\int_0^{t_1} Kv\, du = \int_0^{\mu} Kv\, du + \int_{\mu}^{t_1} Kv\, du$$

$$= \frac{\alpha_1\beta D_0^{1-\gamma}}{(2-\gamma)}[(\gamma-1)\mu^{2-\gamma} + (2-\gamma)\mu^{1-\gamma}t_1], \gamma \neq 2 \quad (17)$$

The Total average inventory cost C is given by

C = Inventory cost + Deterioration cost + Production Cost

$$C = \frac{1}{t_2}\Bigg[(\beta-1)D_0c_1\left(\frac{\mu^3}{6} - \frac{\theta\mu^5}{40} - \frac{\theta^2\mu^7}{48\times 7}\right) + (\beta-1)D_0\mu c_1\left(\frac{{t_1}^2}{2} - \frac{\mu t_1}{2} - \frac{\theta {t_1}^4}{12} + \frac{\theta\mu^4}{24} - \frac{\theta\mu^3 t_1}{24} - \frac{\theta^2 {t_1}^6}{180} + \frac{5\theta^2\mu^6}{720} + \frac{\theta\mu {t_1}^3}{12} + \frac{\theta^2 {t_1}^3\mu^3}{144} - \frac{\theta^2\mu {t_1}^5}{40}\right) + D_0\mu c_1\left(\frac{{t_2}^2}{2} + \frac{{t_1}^2}{2} - t_2t_1 + \frac{\theta {t_2}^4}{12} - \frac{\theta {t_2}^3 t_1}{6} - \frac{\theta {t_1}^4}{12} + \frac{\theta^2 {t_2}^6}{36} - \frac{\theta^2 {t_1}^6}{180} - \frac{\theta^2 {t_2}^5 t_1}{20} + \frac{\theta t_2 {t_1}^3}{6} + \frac{\theta^2 {t_2}^3 {t_1}^3}{36}\right) + \frac{1}{2}D_0\beta\mu\, c_3(2t_1 - \mu) - \frac{1}{2}D_0\mu c_3(2t_2 - \mu) + \frac{\alpha_1\beta D_0^{1-\gamma}}{(2-\gamma)}\{(\gamma-1)\mu^{2-\gamma} + (2-\gamma)\mu^{1-\gamma}t_1\}\Bigg] \quad (18)$$

$$c_1 = c_{01} + \frac{c_1'}{n^{\alpha_2}}$$

Where c_1 is continuously deceases over n since $\frac{dc_1}{dn} < 0, n > 0$

$$c_3 = c_{03} + \frac{c_3'}{n^{\alpha_2}}$$

Where c_3 is continuously deceases over n since $\frac{dc_3}{dn} < 0, n > 0$

Optimum values of t_1 and t_2 for minimum average cost C are the solutions of the equations

$$\frac{\partial C}{\partial t_1} = 0 \; and \; \frac{\partial C}{\partial t_2} = 0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 C}{\partial t_1^2} > 0, \frac{\partial^2 C}{\partial t_2^2} > 0 \text{ and } \frac{\partial^2 C}{\partial t_1^2}\frac{\partial^2 C}{\partial t_2^2} - \left(\frac{\partial^2 C}{\partial t_1 \partial t_2}\right)^2 > 0$$

$$\frac{\partial C}{\partial t_1} = 0 \; and \; \frac{\partial C}{\partial t_2} = 0 \text{ gives}$$

$$(\beta-1)D_0\mu c_1\left[t_1-\frac{\mu}{2}-\frac{\theta t_1^3}{3}-\frac{\theta\mu^3}{24}-\frac{\theta^2 t_1^5}{30}+\frac{\theta\mu t_1^2}{4}+\frac{\theta^2 t_1^2\mu^3}{48}-\frac{\theta^2\mu t_1^4}{8}\right]+D_0\mu c_1\left[t_1-t_2-\frac{\theta t_2^3}{6}-\frac{\theta t_1^3}{3}-\frac{\theta^2 t_1^5}{30}-\frac{\theta^2 t_2^5}{20}+\frac{\theta t_2 t_1^2}{2}+\frac{\theta^2 t_2^3 t_1^2}{12}\right]+D_0\beta\mu c_3+\alpha_1\beta D_0^{1-\gamma}\mu^{1-\gamma}=0 \quad (19)$$

And

$$D_0\mu c_1\left[t_2-t_1+\frac{\theta t_2^3}{3}-\frac{\theta t_2^2 t_1}{2}+\frac{\theta^2 t_2^5}{6}-\frac{\theta^2 t_2^4 t_1}{4}+\frac{\theta t_1^3}{6}+\frac{\theta^2 t_2^2 t_1^3}{12}\right]-D_0\mu c_3-C=0 \quad (20)$$

Case II ($t_1 \le \mu \le t_2$)

The stock level initially is zero production begins just after t=0, continues upto t=$t_1$and stops as soon as the stock level becomes p at t=t_2. Then the inventory level decreases due to both demand and deterioration till it become again zero at t=t_2. Then, the cycle repeats.

Let I (t) be the inventory level of the system at any time t $(0 \le t \le t_2)$. The differential equations governing the system in the interval [0, t_2] are given by

$$\frac{dI(t)}{dt}+\theta t I(t)=K-f(t), \qquad 0\le t\le t_1 \quad (21)$$

With the condition I (0) =0, I(t_1)=p;

$$\frac{dI(t)}{dt}+\theta t I(t)=-f(t), \qquad t_1\le t\le \mu \quad (22)$$

With the condition $I(t_1)$ =p;

$$\frac{dI(t)}{dt}+\theta t I(t)=-f(t), \qquad \mu\le t\le t_2 \quad (23)$$

With the condition I (t_2) =0;

Using ramp type function f (t), equation (21),(22),(23) become respectively

$$\frac{dI(t)}{dt}+\theta t I(t)=(\beta-1)D_0 t, \qquad 0\le t\le t_1 \quad (24)$$

With the condition $I(0)=0$ *and* $I(t_1)$=p;

$$\frac{dI(t)}{dt}+\theta t I(t)=-D_0 t, \qquad t_1\le t\le \mu \quad (25)$$

With the condition I (t_1) =p;

$$\frac{dI(t)}{dt}+\theta t I(t)=-D_0\mu, \qquad \mu\le t\le t_2 \quad (26)$$

With the condition $I(t_2)=0$

The solution of equation(24)is given by

$$I(t)=(\beta-1)D_0\left(\frac{t^2}{2}+\frac{\theta t^4}{8}+\frac{\theta^2 t^6}{24}\right)e^{-\frac{\theta t^2}{2}} \qquad 0\le t\le t_1 \quad (27)$$

Using the condition $I(t_1)=p$, we have

$$p=(\beta-1)D_0\left(\frac{t_1^2}{2}+\frac{\theta t_1^4}{8}+\frac{\theta^2 t_1^6}{24}\right)e^{-\frac{\theta t_1^2}{2}} \quad (28)$$

Therefore the solution of the equation (25) is

$$I(t)\,e^{\frac{\theta t^2}{2}}=-D_0\left[\frac{t^2}{2}+\frac{\theta t^4}{8}+\frac{\theta^2 t^6}{24}\right]+pe^{\frac{\theta t_1^2}{2}}+D_0\left[\frac{t_1^2}{2}+\frac{\theta {t_1}^4}{8}+\frac{\theta^2 t_1^6}{24}\right]$$

$$I(t)=-D_0e^{-\frac{\theta t^2}{2}}\left[\frac{t^2}{2}+\frac{\theta t^4}{8}+\frac{\theta^2 t^6}{24}\right]+\beta D_0e^{-\frac{\theta t^2}{2}}\left[\frac{t_1^2}{2}+\frac{\theta {t_1}^4}{8}+\frac{\theta^2 {t_1}^6}{24}\right] \quad (29)$$

Using the condition $I(t_2)=0$, the solution of equation (26) is given by

$$I(t)=D_0\mu\left[(t_2-t)+\frac{\theta}{6}({t_2}^3-t^3)+\frac{\theta^2}{20}({t_2}^5-t^5)\right]e^{-\frac{\theta t^2}{2}}, \mu\le t\le t_2 \quad (30)$$

Total inventory in [o, t_2] is

$$\int_0^{t_2} I(t)\,dt=\int_0^{t_1} I(t)dt+\int_{t_1}^{\mu} I(t)+\int_{\mu}^{t_2} I(t)dt$$

$$\int_0^{t_1} I(t)dt=(\beta-1)D_0\left(\frac{{t_1}^3}{6}-\frac{\theta {t_1}^5}{40}-\frac{\theta^2 {t_1}^7}{7\times 48}\right)$$

$$\int_{t_1}^{\mu} I(t)dt=D_0\left[\frac{(t_1^3-\mu^3)}{6}+\frac{\theta}{40}(\mu^5-t_1^5)+\frac{\theta^2}{7\times 48}(-t_1^7+\mu^7)\right]+\beta D_0\mu\left(\frac{t_1^2}{2}+\frac{\theta t_1^4}{8}+\frac{\theta^2 t_1^6}{24}-\frac{\theta\mu^2 t_1^2}{12}-\frac{\theta^2\mu^2 t_1^4}{48}\right)+\beta D_0\left(-\frac{t_1^3}{2}-\frac{\theta t_1^5}{24}-\frac{\theta^2 t_1^7}{48}\right)$$

$$\int_{\mu}^{t_2} I(t)dt = D_0\,\mu\,[\left(\frac{t_2^2}{2} - \mu t_2 + \frac{\mu^2}{2}\right) + \theta\left(\frac{t_2^4}{12} - \frac{t_2^3\mu}{6} + \frac{t_2\mu^3}{6} - \frac{\mu^4}{12}\right) + \theta^2(\frac{t_2^6}{36} - \frac{t_2^5\mu}{20} + \frac{t_2^3\mu^3}{36} - \frac{\mu^6}{180})]$$

$$\int_0^{t_2} \mathrm{I(t)dt} = (\beta-1)D_0(\frac{t_1^3}{6} - \frac{\theta t_1^5}{40} - \frac{\theta^2 t_1^7}{7\times 48}) + D_0\left[\left\{\frac{(t_1^3-\mu^3)}{6} + \frac{\theta}{40}(\mu^5 - t_1^5) + \frac{\theta^2}{7\times 48}(-t_1^7 + \mu^7)\right\} + \beta\mu\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8} + \frac{\theta^2 t_1^6}{24} - \frac{\theta\mu^2 t_1^2}{12} - \frac{\theta^2\mu^2 t_1^4}{48}\right) + \beta\left(\frac{-t_1^3}{2} - \frac{\theta t_1^5}{24} - \frac{\theta^2 t_1^7}{48}\right)\right] + D_0\mathrm{u}\left[\left(\frac{t_2^2}{2} - \mu t_2 + \frac{\mu^2}{2}\right) + \theta\left(\frac{t_2^4}{12} - \frac{t_2^3\mu}{6} + \frac{t_2\mu^3}{6} - \frac{\mu^4}{12}\right) + \theta^2\left(\frac{t_2^6}{36} - \frac{t_2\mu}{20} + \frac{t_2^3\mu^3}{36} - \frac{\mu^6}{180}\right)\right] \quad (31)$$

Total number of deteriorated items in $[0, t_2]$ is given by

Production in $[0, t_1]$-Demand in $[0, \mu]$ –Demand in $[\mu, t_2]$

$$=\beta D_0\int_0^{t_1}\mathrm{tdt} - D_0\int_0^{\mu}\mathrm{tdt} - D_0\mu\int_{\mu}^{t_2}\mathrm{dt}$$

$$= \beta D_0\frac{t_1^2}{2} + D_0\frac{\mu}{2}(\mu - 2t_2) \quad (32)$$

Hence, the Production cost in $[0, t_1]$ is given by

$$\int_0^{t_1} Kvdu = \int_0^{t_1}\frac{\alpha_1\beta}{R^{\gamma-1}}du \qquad [using\ 16]$$

$$=\int_0^{t_1}\frac{\alpha_1\beta}{D_0^{\gamma-1}u^{\gamma-1}}du$$

$$= \frac{\alpha_1\beta D_0^{1-\gamma}t_1^{2-\gamma}}{(2-\gamma)}, \gamma \neq 2 \quad (33)$$

From (31),(32),(33) and total average inventory cost C of the system is

$$\mathrm{C}=\frac{1}{t_2}[(\beta-1)D_0c_1\left(\frac{t_1^3}{6} - \frac{\theta {t_1}^5}{40} - \frac{\theta^2 {t_1}^7}{7\times 48}\right) + D_0c_1\left\{\frac{(t_1^3-\mu^3)}{6} + \frac{\theta}{40}(\mu^5 - {t_1}^5) + \frac{\theta^2}{7\times 48}(-{t_1}^7 + \mu^7)\right\} + \beta\mu D_0C_1\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8} + \frac{\theta^2 t_1^6}{24} - \frac{\theta\mu^2 {t_1}^2}{12} - \frac{\theta^2\mu^2 {t_1}^4}{48}\right) + \beta D_0c_1\left(\frac{-t_1^3}{2} - \frac{\theta {t_1}^5}{24} - \frac{1}{24}\theta^2 {t_1}^7\right) + D_0\mu c_1\{(\frac{{t_2}^2}{2} - \mu t_2 + \frac{\mu^2}{2}) + \theta(\frac{{t_2}^4}{12} - \frac{{t_2}^3\mu}{6} + \frac{{t_2}\mu^3}{6}\frac{-\mu^4}{12}) + \theta^2(\frac{{t_2}^6}{36} - \frac{{t_2}^5\mu}{20} + \frac{{t_2}^3\mu^3}{36} - \frac{\mu^6}{180})\} + \beta D_0c_3\frac{{t_1}^2}{2} + \frac{D_0c_3\mu}{2}(\mu - 2t_2) + \frac{\alpha_1\beta {D_0}^{1-\gamma}}{(2-\gamma)}{t_1}^{2-\gamma}] \quad (34)$$

$$c_1 = c_{01} + \frac{c_1'}{n^{\alpha_2}}$$

Where c_1 is continuously deceases over n since $\frac{dc_1}{dn} < 0$,n> 0

$$c_3 = c_{03} + \frac{c_3'}{n^{\alpha_2}}$$

Where c_3 is continuously deceases over n since $\frac{dc_3}{dn} < 0$,n> 0

Optimum values of t_1 and t_2 for minimum average cost are obtained as in case I which gives

$$\frac{\partial c}{\partial t_1} = 0$$

$$(\beta-1)\,D_0c_1\left(\frac{3{t_1}^2}{6} - \frac{5\theta {t_1}^4}{40} - \frac{7\theta {t_1}^6}{7\times 48}\right) + D_0c_1\left(\frac{3{t_1}^2}{6} - \frac{5{t_1}^4\theta}{40} - \frac{7\theta^2 {t_1}^6}{7\times 48}\right) + \beta\mu Doc_1\left(\frac{2t_1}{2} + \frac{4\theta {t_1}^3}{8} + \frac{6\theta^2 {t_1}^5}{24} - \frac{2\theta\mu^2 t_1}{12} - \frac{4\theta\mu^2 {t_1}^3}{48}\right) + \beta D_0c_1\left(-\frac{3{t_1}^2}{2} - \frac{5\theta {t_1}^4}{40} - \frac{7\theta^2 {t_1}^6}{48}\right) + \beta D_0c_3t_1 + \frac{\alpha_1\beta D_0^{1-\gamma}(2-\gamma)t_1^{1-\gamma}}{(2-\gamma)} = 0$$

and

$$\frac{\partial c}{\partial t_2} = 0$$

$$\mu D_0c_1\left[(t_2-\mu) + \theta\left(\frac{4{t_2}^3}{12} - \frac{3{t_2}^2\mu}{6} + \frac{\mu^3}{6}\right) + \theta^2\left(\frac{6{t_2}^5}{36} - \frac{5{t_2}^4\mu}{20} + \frac{3{t_2}^2\mu^3}{36}\right)\right] - D_0\mu c_3 - C = 0$$

Reducing form

$$(\beta-1)\,D_0c_1\left(\frac{{t_1}^2}{2} - \frac{\theta {t_1}^4}{8} - \frac{\theta {t_1}^6}{48}\right) + D_0c_1\left(\frac{{t_1}^2}{2} - \frac{{t_1}^4\theta}{8} - \frac{\theta^2 {t_1}^6}{48}\right) + \beta\mu D_0c_1\left(t_1 + \frac{\theta {t_1}^3}{2} + \frac{\theta^2 {t_1}^5}{4} - \frac{\theta\mu^2 t_1}{6} - \frac{\theta^2\mu^2 {t_1}^3}{12}\right) + \beta D_0c_1\left(\frac{-3{t_1}^2}{2} - \frac{5\theta {t_1}^4}{24} - \frac{7\theta^2 {t_1}^6}{24}\right) + \beta D_0c_3t_1 + \alpha_1\beta {D_0}^{1-\gamma}{t_1}^{1-\gamma} = 0 \quad (35)$$

And

$$\frac{\partial c}{\partial t_2} = 0$$

$$D_0\mu c_1(t_2-\mu) + \theta\left(\frac{{t_2}^3}{3} - \frac{{t_2}^2\mu}{2} + \frac{\mu^3}{6}\right)\theta^2\left(\frac{{t_2}^5}{6} - \frac{{t_2}^4\mu}{4} + \frac{{t_2}^3}{3} - \frac{{t_2}^2\mu^3}{12}\right) - D_0\mu c_3 - C = 0 \quad (36)$$

NUMERICAL EXAMPLE

Let us consider the inventory system with following data for case I ($\mu \leq t_1 \leq t_2$)
$\gamma = 2.2, D_0 = 10, \mu = 0.5, \beta = 2.8, \theta = 0.4, \alpha_1 = 2.1, c_{01} = 4, c_1' = 2, n = 2, \alpha_2 = 1, c_{03} = 2, c_3' = 1$
Output Results Are

$$t_1 = 1.37247, t_2 = 1.83476, \qquad T.C = 27.5149$$

Table. 1. SENSITIVITY ANALYSIS: The sensitive analysis of the key parameter γ, D_0, θ

Parameters		t_1	t_2	T.C
γ	2.3	1.37674	1.85429	26.5028
	2.4	1.37851	1.86345	26.0439
	2.5	1.37934	1.86862	25.7948
	2.6	1.37973	1.87184	25.6458
D_0	11	1.37371	1.84347	29.7988
	12	1.37464	1.84993	32.1318
	13	1.37535	1.85485	34.5008
	14	1.37592	1.85867	36.8969
θ	0.39	1.37212	1.8543	27.2083
	0.38	1.37160	1.87484	26.8957
	0.37	1.37088	1.89648	26.5768
	0.36	1.36995	1.91934	26.2516

OBSERVATIONS

1. t_1 & t_2 increase while T.C decreases with the increase in value of the parameter γ.
2. t_1 & t_2 increase and T.C also increases with the increase in value of the parameter D_0.
3. t_1 &T.C decrease while t_2 increases with the decrease in value of the parameter θ.

CASE 2 ($t_1 \leq \mu \leq t_2$)
$\gamma = 1.3, D_0 = 20, \mu = 1.2, \beta = 2, \theta = 1.3, \alpha_1 = 2, c_{01} = 3, c_1' = 4, n = 2, \alpha_2 = 1, c_{03} = 1, c_3' = 1$
Output results are

$$t_1 = 1.10592, t_2 = 1.45479, \qquad T.C = 6.34804$$

Table. 2. SENSITIVITY ANALYSIS: The sensitive analysis of the key parameter γ, D_0, θ

Parameters		t_1	t_2	T.C
γ	1.4	1.10674	1.4558	6.59515
	1.5	1.10733	1.45652	6.7708
	1.6	1.10776	1.45696	6.87806
	1.7	1.10808	1.45707	6.90401
D_0	21	1.10611	1.45522	6.77615
	22	1.10628	1.45561	7.20275
	23	1.10643	1.45596	7.62799
	24	1.10657	1.45627	8.052
θ	1.29	1.10839	1.45633	6.5412
	1.28	1.11087	1.45788	6.735
	1.27	1.11336	1.45944	6.92946
	1.26	1.11585	1.46102	7.12452

OBSERVATIONS

1. t_1 & t_2 increase and T.C also increases with the increase in value of the parameter γ.
2. t_1 &t_2 increase and T.C also increases with the increase in value of the parameter Do.
3. t_1 & t_2 increase and T.C also increases with the decrease in value of the parameter θ.

CONCLUSION

In this study, an EOQ model with ramp type demand rate and unit production cost under learning phenomenon has been developed. The quality and quantity of goods decrease in course of time due to deterioration it is a natural phenomena.

Hence consideration of time dependent deterioration function defines a significant meaning of perishable, volatile and failure of any kind of item. A mathematical model has been developed to determine the optimal ordering policy cost which minimizes the present worth of total optimal cost. Thus the model concludes with numerical examples.

Equation (19) and (20) are non- linear equation in t_1 and t_2. These simultaneous non-linear equations can be solved for suitable choice of the parameters c_1, c_3, $\beta, \mu, n, D_0, \alpha_1, \alpha_2$ and γ ($\neq 2$). If t_1^* and t_2^* are the solution of (19) and (20) for Case I, the corresponding minimum cost $c^*(t_1, t_2)$ can be obtained from (18). It is very difficult to show analytically whether the cost function C (t_1, t_2) is convex. That is why; C (t_1, t_2) may not be global minimum. If C (t_1, t_2) is not convex, then C (t_1, t_2) will be local minimum.

Similarly, solution of equations (35) and (36) for Case II can be obtained corresponding minimum cost C (t_1, t_2) can be obtained from (34).

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